

## 1.6 Systems of Equations

Consider the system below:

$$3x - y = 7$$

$$x - 2y = 4$$

Is the ordered pair  $(2, -1)$  a solution? Yes

$$3(2) - (-1) = 7$$

$$2 - (-2)(-1) = 4$$

$$(3, 2)$$

$$3(3) - 2 = 7$$

$$3 - 2(2) \neq 4$$

Is the ordered pair  $(3, 2)$  a solution?

## Ways to solve systems

- Graphing ✓
- Algebraically
  - Substitution ·
  - Elimination ·

Solve the system by Substitution.  $\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$

$$\begin{array}{r} 2x + y = 7 \\ -2x \end{array}$$

$$y = 7 - 2x$$

$$\begin{aligned} y &= 7 - 2(4) \\ &= 7 - 8 \\ &= -1 \end{aligned}$$

$$(4, -1)$$

$$x - 2(7 - 2x) = 6$$

$$x - 14 + 4x = 6$$

$$5x - 14 = 6$$

$$5x = 20$$

$$x = 4$$

Solve the system by Substitution.

$$\begin{cases} -7x - 2y = -13 \\ x - 2y = 11 \end{cases}$$

$$x - 2y = 11$$

$$x = 11 + 2y$$

$$11 + 2(-4)$$

$$11 - 8$$

$$x = 3$$

$$(3, -4)$$

$$-7(11 + 2y) - 2y = -13$$

$$-77 - 14y - 2y = -13$$

$$-77 - 16y = -13$$

$$+77 \qquad +77$$

$$-16y = 64$$

$$y = -4$$

Solve the system by Substitution.  $\begin{cases} -2x + 6y = 6 \\ -7x + 8y = -5 \end{cases}$

$$\begin{array}{r} -2x + 6y = 6 \\ \quad -6y \quad -6y \\ \hline \end{array}$$

$$\frac{-2x}{-2} = \frac{6-6y}{-2}$$

$$x = -3 + 3y$$

$$\begin{array}{r} -3 + 3(2) \\ -3 + 6 \\ \hline 3 \end{array}$$

$(3, 2)$

$$-7(-3 + 3y) + 8y = -5$$

$$21 - 21y + 8y = -5$$

$$21 - 13y = -5$$

$$-13y = -26$$

$$y = 2$$

Solve the system by elimination  $\begin{cases} 3x - 2y = -2 \\ 5x - 6y = 10 \end{cases}$

$$\begin{aligned} 3(-4) - 2y &= -2 \\ -12 - 2y &= -2 \\ -2y &= 10 \\ y &= -5 \end{aligned}$$

$$(-4, -5)$$

$$\begin{aligned} 9x - 6y &= -6 \\ \underline{-(5x - 6y = 10)} & \\ 4x &= -16 \\ x &= -4 \end{aligned}$$

Solve the system by elimination  $\begin{cases} 4x - 3y = 1 \\ 5x - 9y = -4 \end{cases}$

$$4(1) - 3y = 1$$

$$4 - 3y = 1$$

$$-3y = -3$$

$$y = 1$$

$$(1, 1)$$

$$12x - 9y = 3$$

$$-5x + 7y = -4$$

$$7x = 7$$

$$x = 1$$



Solve the system by elimination

$$\begin{cases} (4x - 3y = 9) & \times 2 & 8x - 6y = 18 \\ (7x + 2y = -6) & \times 3 & 21x + 6y = -18 \\ \hline & & 29x = 0 \\ & & x = 0 \end{cases}$$

$(0, -2)$

$$\begin{array}{l}
 18) \quad (x - 3y = 1) \quad 2x - 6y = 2 \\
 2x - y = 7 \quad \underline{2x - y = 7} \\
 \quad \quad \quad -5y = -5 \\
 \quad \quad \quad y = 1
 \end{array}$$

$$\begin{array}{l}
 x - 3(1) = 1 \quad (4, 1) \\
 x - 3 = 1 \\
 x = 4
 \end{array}$$

$$\begin{array}{l}
 19) \quad x + 2y = -4 \\
 2(3x - y = -5) \\
 \quad \quad \quad -2 + 2y = -4 \\
 \quad \quad \quad 2y = -2 \\
 \quad \quad \quad y = -1
 \end{array}$$

$$\begin{array}{l}
 x + 2y = -4 \\
 \underline{6x - 2y = -10} \\
 7x = -14 \\
 x = -2 \\
 (-2, -1)
 \end{array}$$

Solve the system by elimination  $\begin{cases} 3x - 4y = -9 \\ 5x + 3y = 14 \end{cases}$

$$\begin{cases} \textcircled{1} 2x - 4y + z = 10 \\ \textcircled{2} (x + 2y - z = 1)^2 \\ \textcircled{3} -x - 3y + 2z = 0 \end{cases}$$

$$\begin{array}{r} E_1 + E_2 \\ 2x - 4y + z = 10 \\ (+) x + 2y - z = 1 \\ \hline \end{array}$$

$$\textcircled{4} 3x - 2y = 11$$

$$E_2 + 5E_3$$

$$\begin{array}{r} 2x + 4y - 2z = 2 \\ (+) -x - 3y + 2z = 0 \\ \hline \end{array}$$

$$\textcircled{5} x + y = 2$$

$$3x - 2y = 11 = 3x - 2y = 11$$

$$2(x + y = 2) = (+) 2x + 2y = 4$$

$$\begin{array}{r} 5x = 15 \\ x = 3 \end{array}$$

$$3 + y = 2$$

$$y = -1$$

$$(3, -1, 0)$$

$$2x - 4y + z = 10$$

$$2(3) - 4(-1) + z = 10$$

$$6 + 4 + z = 10$$

$$z = 0$$

$$\textcircled{1} 2x + y + z = -2$$

$$\textcircled{2} 2x - y + 3z = 6$$

$$\textcircled{3} 3x - 5y + 4z = 7$$

$$\frac{E_1 + E_2}{2x + y + z = -2}$$

$$\textcircled{+} \frac{2x - y + 3z = 6}{2x - y + 3z = 6}$$

$$\textcircled{4} 4x + 4z = 4$$

$$E_1 + E_3$$

$$10x + 5y + 5z = -10$$

$$\textcircled{+} \frac{3x - 5y + 4z = 7}{3x - 5y + 4z = 7}$$

$$\textcircled{5} 13x + 9z = -3$$

$$4x + 4z = 4 = x + z = 1$$

$$13x + 9z = -3 \quad 13x + 9z = -3$$

$$2x + y + z = -2$$

$$2(-3) + y + 4 = -2$$

$$-6 + y + 4 = -2$$

$$y - 2 = -2$$

$$y = 0$$

$$-3 + z = 1$$

$$z = 4$$

$$(-3, 0, 4)$$

$$13x + 9z = -3$$

$$\textcircled{-} \frac{9x + 9z = 9}{9x + 9z = 9}$$

$$4x = -12$$

$$x = -3$$

$$z = 4$$

$$\begin{cases} x+3y-3z=12 \\ 3x-y+4z=0 \\ -x+2y-z=1 \end{cases}$$

$(3, 1, -2)$